

---

Retrospective Theses and Dissertations

---

1988

## Significant measurements of a multiple target tracking system utilizing munkre's algorithm as a correlation scheme

Nathalie Anna Day  
University of Central Florida

 Part of the [Engineering Commons](#)

Find similar works at: <https://stars.library.ucf.edu/rtd>

University of Central Florida Libraries <http://library.ucf.edu>

This Masters Thesis (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of STARS. For more information, please contact [STARS@ucf.edu](mailto:STARS@ucf.edu).

---

### STARS Citation

Day, Nathalie Anna, "Significant measurements of a multiple target tracking system utilizing munkre's algorithm as a correlation scheme" (1988). *Retrospective Theses and Dissertations*. 4271.  
<https://stars.library.ucf.edu/rtd/4271>

SIGNIFICANT MEASUREMENTS OF A MULTIPLE  
TARGET TRACKING SYSTEM UTILIZING MUNKRE'S ALGORITHM  
AS A CORRELATION SCHEME

BY

NATHALIE ANNA DAY  
B.S.E., University of Central Florida, 1985

THESIS

Submitted in partial fulfillment of the requirements  
for the degree of Master of Science in Engineering  
in the Graduate Studies Program  
of the College of Engineering  
University of Central Florida  
Orlando, Florida

Summer Term  
1988

## ABSTRACT

This thesis presents and discusses the principles of multiple target tracking. A simulation written in Turbo Pascal provides the results of using a modified version of Munkre's algorithm for correlating targets with observations. The number and types of measurements necessary to obtain acceptable results are examined. The measurements under scrutiny are range, range rate, azimuth angle and elevation angle. A track-while-scan system is assumed and the nearest neighbor correlation scheme as well as rectangular gating are used for association.

## ACKNOWLEDGEMENTS

I would like to take this space to extend my deepest thanks and appreciation to my mom and dad for all their support in getting me this far in life. I would also like to thank Alex, my fiance, for his help and understanding during the past year. Special thanks are in order for Dr. Wahid for her efforts in helping me with this thesis. Mark Owens and Dave Freedman deserve recognition for their excellent technical support. I can't forget the hand that fed me for four semesters, the Industrial Associates Program sponsored by Martin Marietta, and its coordinators, John Gill and Dale Ann Tako. Last but not least, I think my academic and work friends deserve some recognition. Little might they know that the times we spent together dancing and partying worked wonders in easing the stress caused by writing a thesis. Few words can express the gratitude I feel towards everyone who helped me in preparing for this thesis... therefore, I offer but a few choice words: Thanks to everyone!



## TABLE OF CONTENTS

LIST OF TABLES . . . . .	v
LIST OF FIGURES . . . . .	vi
INTRODUCTION . . . . .	1
CHAPTER ONE - DETECTION . . . . .	4
CHAPTER TWO - FLIGHT GEOMETRY. . . . .	13
CHAPTER THREE - DEFINING THE TARGET. . . . .	21
CHAPTER FOUR - DEFINING THE OBSERVATION . . . . .	27
CHAPTER FIVE - GATING . . . . .	30
CHAPTER SIX - AN EXTENSION TO MUNKRE'S ALGORITHM FOR THE DYNAMIC CASE . . . . .	34
CHAPTER SEVEN - SIGNIFICANT MEASUREMENTS OF AN MTT SYSTEM . . . . .	56
CHAPTER EIGHT - SOFTWARE DESIGN OF AN MTT SIMULATION . . .	69
CHAPTER NINE - CONCLUSION . . . . .	77
REFERENCES . . . . .	79

## LIST OF TABLES

1. RANGE MEASUREMENTS FOR FOUR TARGETS . . . . .	43
2. RANGE MEASUREMENTS FOR FOUR OBSERVATIONS . . . . .	44
3. RANGE AND AZIMUTH MEASUREMENTS FOR ONE SCAN . . . . .	53
4. RADAR SYSTEM PARAMETERS USED IN THIS EXAMPLE. . . . .	57
5. SIGNIFICANT MEASUREMENTS OF AN MTT SYSTEM RANKED IN ORDER OF IMPORTANCE . . . . .	67

## LIST OF FIGURES

1.1	Example of a Thresholder .. . . .	5
2.1	Flight Geometry .. . . .	14
2.2	XY Projection of the Configuration . . . . .	17
3.1	Threshold Detector Controlling the Targets' Lives	25
5.1	Example of Gating and Correlation . . . . .	31
6.1	Sample Matrix for Munkre's Algorithm . . . . .	35
7.1	Tally of Number of Observations and Targets During 100 seconds . . . . .	58
7.2	Correlation Plot 1 . . . . .	60
7.3	Correlation Plot 2 . . . . .	61
7.4	Correlation Plot 3 . . . . .	62
7.5	Correlation Plot 4 . . . . .	63
7.6	Correlation Plot 5 . . . . .	64
7.7	Correlation Plot 6 . . . . .	65
7.8	Correlation Plot 7 . . . . .	66
8.1	Block Diagram of the Simulation . . . . .	70

## INTRODUCTION

Soaring gracefully across a sky whose ocean blue hues are complimented by puffs of pure white clouds is a jet fighter in pursuit of an enemy hundreds of kilometers away. The serenity of the heavens is barely disturbed by this sole man-made intruder as it speeds across the sky. The craft is guided by the echoes of radio waves bouncing off the target and increasing in intensity as the iron bird closes in on its prey. As the aircraft approaches its target, the clouds are no longer as effective in shielding the enemy by absorbing its waves. The enemy's proximity to the aircraft overrides any attenuation to its signal that was made by the atmosphere. The aircraft is on the target's tail and at this point ... well... the target is history. And the fighter soars off over the horizon in search of new prey to feed its insatiable hunger.

The above story seems like something out of a science fiction movie. How can an aircraft hundreds of kilometers away zero in on a target? It's quite simple actually! It uses the principles of radar. In short, a radar sends out a signal and counts on receiving echoes that have bounced off of targets and returned to the radar. By keeping track of the time it takes for a signal to return and measuring the

strength of the signal, the distance from the radar to the target can be estimated quite accurately.

Radar Engineering has made many advances in the last century. There are a large number of different types of radars that have been designed to match a variety of needs. The system designed here is classified as a multiple target tracking (MTT) system. This implies that it has the ability to track more than one target at any given time. A track-while-scan (TWS) system, which is a special case of the MTT system is utilized. This system scans a search volume while simultaneously receiving data at regular intervals. Another characteristic of the TWS system is that targets must be within the antenna's search volume in order to be tracked. The TWS system under study uses a single sensor to monitor its environment and scans at a constant rate. Once targets have been spotted, the radar must process the observations and match them to a certain target. This is known as association. Nearest neighbor association is assumed. This means that the observation that is closest to a predicted target position becomes correlated to that target. Gating, which is a means of forming boundaries around predicted target positions, is used to eliminate highly unlikely observation-track pairs. If the observation is not within the boundary around a target, it is assumed that the observation is not correlated to the target. Munkre's algorithm associates the observations to the targets. This



algorithm is based on linear algebra and uses matrix theory to come up with a combination of row-column associations that minimizes the total distance between all targets and observations in question. The following chapters elaborate on each of the topics mentioned above.

## CHAPTER ONE - DETECTION

The primary function of a radar system is to detect targets. Environmental factors, system specifications, and component quality affect the radar's ability to achieve this goal. How these elements are modeled into a mathematical equation and how they contribute to the detection process is covered in this chapter.

### Definition of Detection

Before explaining the factors that affect the detection process, it is important to first understand what is meant by detection. Detection is a procedure whereby a signal is processed and categorized as originating from a target or originating from noise. Threshold detection compares the strength of a returned signal, commonly referred to as a return, to an established level. If the return exceeds the level, it is considered to be from a target. Otherwise, it is due to the presence of noise. Figure 1.1 illustrates the threshold detector.

### Defining the Threshold

The threshold level is a function of three basic factors: the probability of detection, the probability of false alarm, and the number of pulses integrated. These



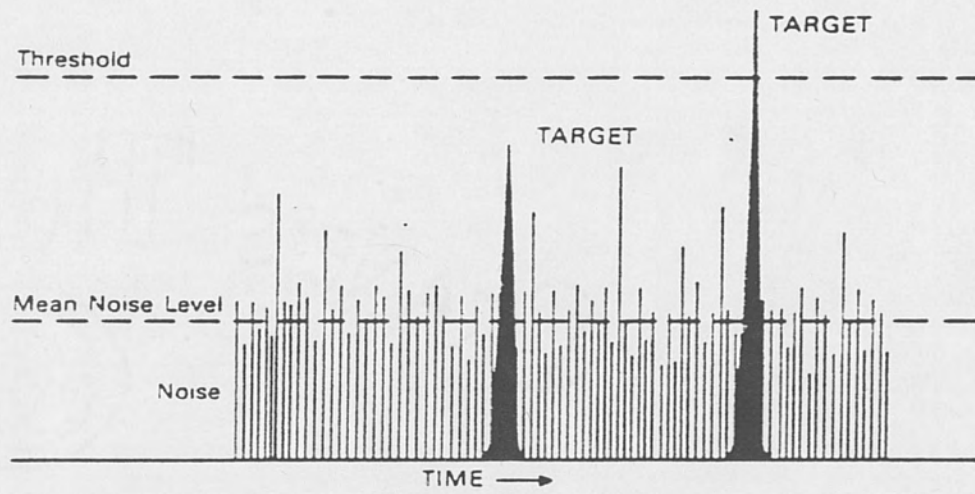


Figure 1.1 Example of a Threshold Detector.

three factors are indigenous to the particular system being designed. The threshold level is also referred to as the desired signal-to-noise ratio.

### Defining the Return

The return, mentioned above, refers to the physical quantity that is measured by the radar. It is a function of the target size, the distance between the target and the radar, the weather conditions, the radar transmit power, the gain, the wavelength and the various losses. Another name for this return is the obtained signal-to-noise ratio.

### Discrimination

Before proceeding, it is important to distinguish between two similar yet different terms: the obtained signal-to-noise ratio (the return) versus the desired signal-to-noise ratio (the threshold). The return is what you actually receive. The threshold is the level you compare the received signals to.

### The Radar Range Equation

The radar range equation is an algorithm reflecting system parameters and target characteristics. It is defined in terms of the obtained signal-to-noise ratio.

$$S/N = \frac{(P_t) (G_t) (G_r) (Trcs) (Lams)}{(4 \cdot \pi)^3 Bif \cdot T \cdot NF \cdot k \cdot L \cdot (R)^4 (0.2 \cdot \text{atmos} \cdot R)}$$

#### Terms Defined:

S/N : Obtained signal-to-noise ratio  
 Pt : Transmitted power (watts)  
 Gt : Gain of the transmitter  
 Gr : Gain of the receiver  
 Trcs : Target cross section (m\*\*2)  
 Lams : Square of the wavelength (m\*\*2)  
 Pi : 3.14159  
 Bif : IF receiver bandwidth (Hz)  
 T : Ambient temperature (Kelvin)  
 NF : Noise Figure of the receiver : figure of merit  
 k : Boltzmann's constant (J/K)  
 L : Losses: Receiver, transmitter, field degradation,  
       : beamshape, elevation beamshape, straddle, CFAR  
 R : Line of sight distance (m) from the ownship to the  
       : target.  
 atmos : Atmospheric attenuation (dB/km)

Following is a brief summary of each component in the radar range equation.

#### Antenna Gain

The gain of an antenna is defined as the ability of an antenna to aim its energy in a particular direction. The

gain is inversely proportional to the beamwidth of the antenna. Sharpening the beam yields greater gain because more energy is concentrated in a smaller area. Gain is also related to the aperture size of the antenna. The larger the aperture size, the narrower the beamwidth will be.

### Target Cross Section

The target cross section is defined as the fictional area intercepting that amount of power which, when scattered equally in all directions, produces an echo at the radar equal to that from the target as defined by Skolnik [1]. The target cross section is one of the most interesting parameters affecting the signal-to-noise ratio. Part of its mystique lies in the fact that for common radar targets such as airplanes, ships and terrains, the cross section does not necessarily bear resemblance to the physical size of the target. Instead, the cross section is a function of the viewing angle and of the radar frequency. Slight changes in viewing aspect results in considerable changes in the cross section. For example [1], a military propeller aircraft such as the AD-4B has a cross section of about 20 square meters in the one to two gigahertz frequency interval, but a one hundred square meter cross section in the thirty to three hundred megahertz interval. Even though there is no standard method for specifying the single-valued cross section of an aircraft, single values are usually cited for specific

targets for use in computing the radar range equation.

#### Mean Noise Power

Noise is electrical energy of random amplitude and phase. It originates in the receiver, is amplified by the gain of the receiver, and then, it propagates through the system. Noise and signal are amplified equally by the receiver. The Noise Figure is used as a figure of merit to compare the noise output of the actual receiver to that of an ideal receiver. Thermal noise, due to the random motion of electrons, is present across the entire frequency spectrum of the receiver. It is proportional to the IF bandwidth and to the ambient temperature. The constant of proportionality is known as Boltzmann's constant. Mean noise power can be summarized as:

$$\text{Mean Noise Power} = (NF) * (k) * (T)$$

Where

k : Boltzmann's constant  $1.38 \times 10^{-23}$  joules/kelvin

NF : Noise Figure

T : Ambient Temperature in degrees Kelvin

#### Losses

A radar system is plagued with various types of losses. Some can be measured by the system while others can only be estimated. The various types of losses that can be encountered in a radar system are discussed below.



Beamshape Loss. When a train of pulses returns from a target, the pulse is assumed to be rectangular in shape. However, this is not an entirely accurate conclusion. In fact that train of pulses is modulated in amplitude by the shape of the antenna beam. If the target is centered in the path of the scanning beam, the loss, known as beamshape loss, is negligible. However, if it is off-centered, this loss can be substantial. To properly handle this effect, it is necessary to compute the probability of detection assuming a modulated train of pulses. Some charts are computed this way while others are not. It is important to be aware of this detail when using such tables.

Field Degradation. All system components are subject to aging. This brings about a loss in performance. If the equipment is not maintained, this loss can be substantial. Factors contributing to field degradation include loose cable connections, water in the transmission lines, degradation of the noise figure, and poor tuning. This loss is minimized by designing radars with built-in performance monitoring equipment. Transmitted power, gain of the receiver and transmitter, noise figure and the shape of the transmitted pulse are characteristics that are most commonly monitored.

Atmospheric Attenuation. As radio waves pass through the atmosphere, they are attenuated by two mechanisms,

absorption and scattering. Absorption occurs when a portion of the traveling energy collides with water and oxygen, and becomes absorbed as heat and consequently lost. Absorption increases with an increase in frequency and decreases with an increase in altitude and a decrease in humidity. Scattering is caused by the presence of particles suspended in the atmosphere. When radio waves collide with these particles, the waves deflect. The most critical scatterer is the rain drop. Of lesser impact to radio waves are snowflakes, clouds, smoke and dust. These effects make the radar extremely sensitive to adverse weather conditions.

Operator Loss. Operator loss is one that proves to be unpredictable. An operator can be alert and well one day, and extremely tired the next. Or, the operator can be inexperienced. How well the operator feels affects the performance of the radar.

#### Chapter Summary

As can be seen, the detection process is a very complicated process involving not only the radar parameters but the environment as well. Targets can be perceived differently due to limitations of the radar and due to the environment. Bad weather, chaff, flocks of birds, and even insects can not only hide the targets from view but can be mistaken for targets. Even when a radar is looking at a target, the operating frequency and the aspect angle can



hinder detection. All of these factors must be considered when setting a threshold such that it is low enough to detect weak signals yet high enough to sift out unwanted noise.

## CHAPTER TWO - FLIGHT GEOMETRY

Two coordinate systems have been chosen to fully define the multiple target tracking (MTT) system. A three dimensional cartesian coordinate system is defined for the ownship and for the targets. The earth is defined as the origin and positions as well as velocities of both targets and the ownship are defined with respect to the earth. In addition, a spherical coordinate system is defined to represent measurements collected by the radar. The spherical coordinates are related to the cartesian coordinates by a set of equations. These equations are presented below.

### Relation Between the Cartesian and Spherical Coordinate Systems

The cartesian and spherical coordinate systems are related by a set of equations. The flight geometry is first defined by the cartesian system and then converted to the spherical coordinate system. Assume that the ownship and targets each have respective position and velocity vectors in the cartesian system, i.e. in the  $x, y$ , and  $z$  directions. The spherical system is defined with respect to the range, range rate, azimuth angle, and elevation angle. If the position and velocity of a target in the cartesian plane is known, the corresponding coordinates in the spherical plane can be calculated.

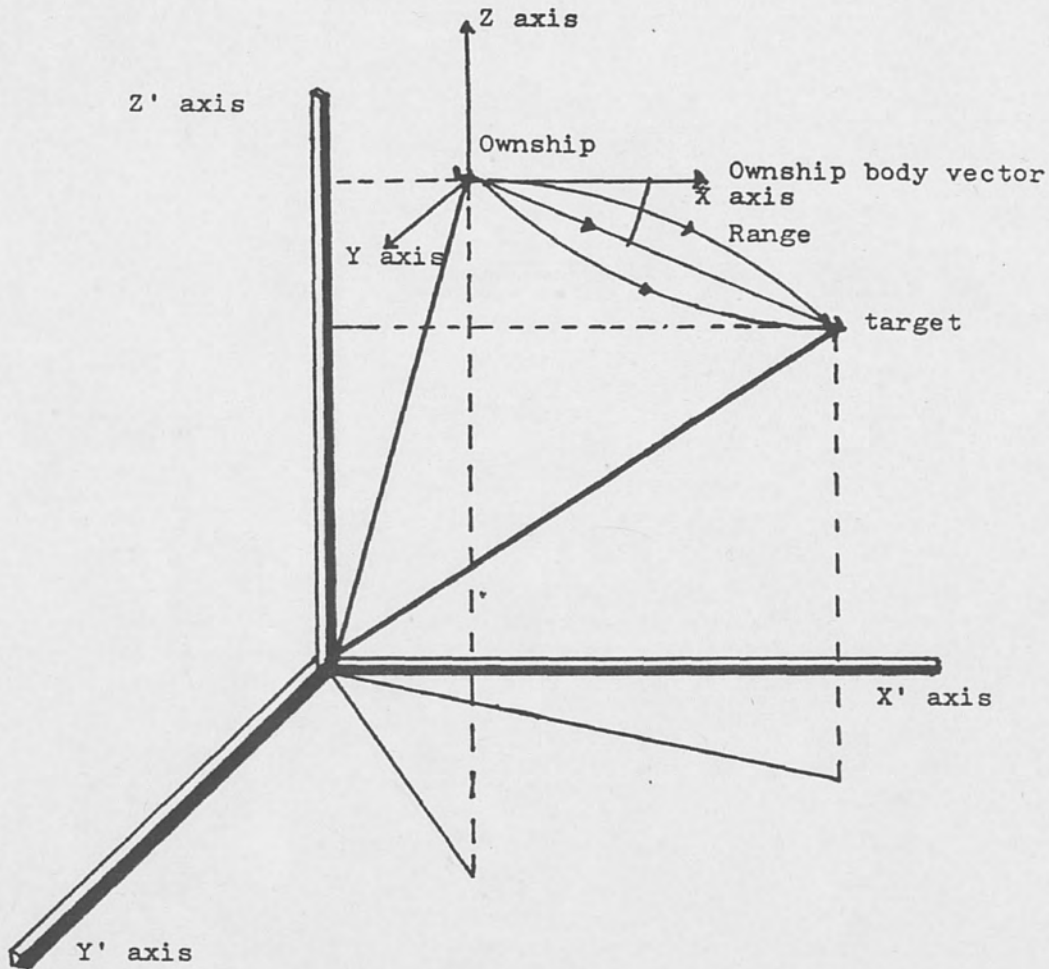


Figure 2.1 Flight Geometry for a Single Target.

Figure 2.1 illustrates the geometry of the multitarget environment. For simplicity, only one target is shown.

### Determination of Range

The linear distance between the ownship and each target is better known as the range. The Pythagorus theorem is used to determine this distance. Let the ownship position be described by the coordinates,  $X_o$ ,  $Y_o$  and  $Z_o$ . Denote the target position by  $X_t$ ,  $Y_t$  and  $Z_t$ . The range is computed as:

$$\text{Range} = [ (X_o - X_t)^2 + (Y_o - Y_t)^2 + (Z_o - Z_t)^2 ]^{1/2}$$

### Computing the Range Rate

The range rate is defined as how fast the target closes in on the ownship. Mathematically, it is defined as the component of the differential velocity vector onto the direction of the line of sight vector. The differential velocity vector is the vector difference between the velocity of the ownship and the velocity of each target.

The component of the the differential velocity vector,  $dV$ , in the direction of the line-of-sight vector,  $R$ , is the range rate and is defined as follows:

### Determination of the Azimuth Angle

The x-y projection of the angle that the position vector of each target makes with the boresite (direction in which the antenna points) vector of the ownship is defined as the azimuth angle. The boresite vector is usually synonymous with the velocity vector of the ownship. However, if the airplane flies sideways and the antenna is aimed in the forward direction, then the body or boresite vector is not in the direction of the velocity vector. In this case, the body vector which is aligned with the direction of the antenna is not the same as the velocity vector which is aimed sideways. Pitch, yaw, and roll come into play at this point. However, for the MTT system described here, it is assumed that the body and velocity vectors are pointing in the same direction. The configuration is shown in Figure 2.2.  $R_{xy}$  is the projection of the line-of-sight vector into the xy plane.  $V_{xy}$  is the projection of the ownship velocity vector into the xy plane.  $Az_R$  is the angle between the vector,  $\vec{R}_{xy}$ , and the x axis:

$$Az_R = \cos^{-1} ( R_x / R_{xy} )$$



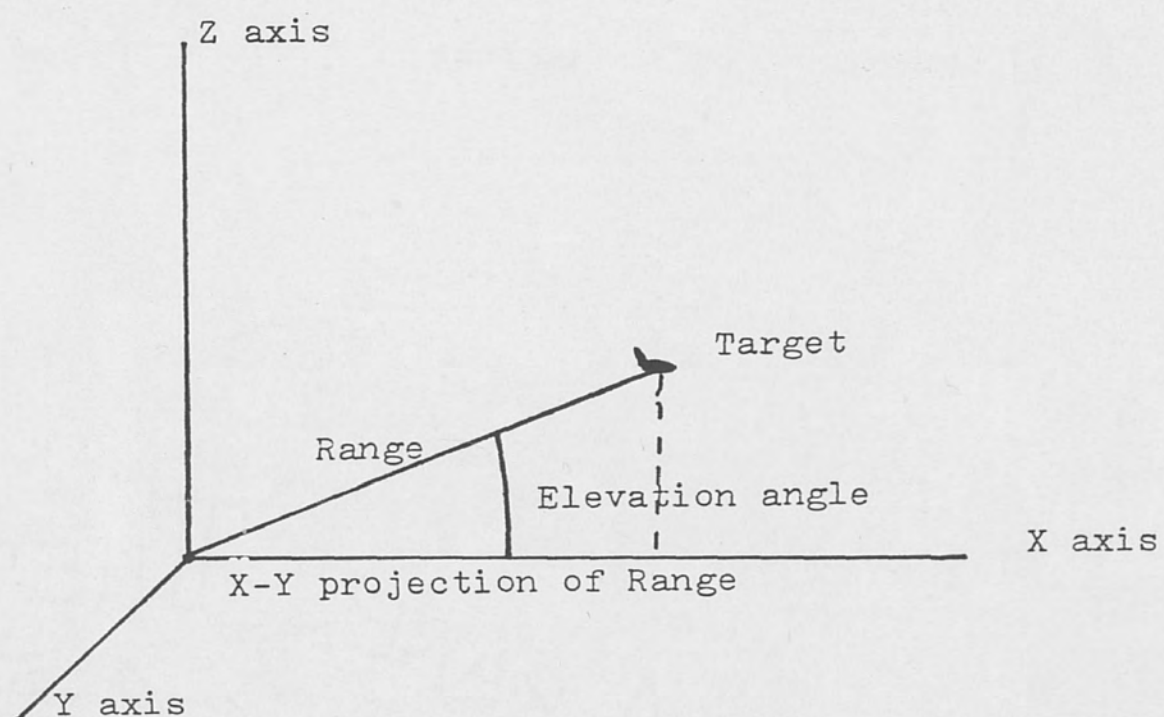


Figure 2.2 XY Projection of the Configuration

The angle between the ownship velocity vector and the x axis is:

$$Az_V = \cos^{-1} ( V_x / V_{xy} )$$

The azimuth angle is the difference of the above-mentioned angles:

$$Az = Az_R - Az_V$$

#### Computing the Elevation Angle

The elevation angle is defined as the angular distance in the vertical plane between the target and the ownship. To determine this angle, the z axis vector is designated as the reference direction. To find the angle the ownship velocity vector forms with the z axis, one performs a dot product with these two vectors. The same is done to find the angle the target forms with the z axis. Subtraction of the two resulting angles will yield the elevation angle of the ownship with respect to the target. The sign of the resulting angle will indicate if the target is above or below the ownship. This is described mathematically as:

Let  $\hat{z} = \hat{az}$  (unit vector in the positive z direction)

$$\vec{O} = V_{xo} \hat{ax} + V_{yo} \hat{ay} + V_{zo} \hat{az}$$



$$\cos ( \text{Elo} ) = \frac{\vec{Z} \cdot \vec{O}}{Z \quad O} = \frac{V_{zo}}{(V_{xo}^2 + V_{yo}^2 + V_{zo}^2)^{1/2}}$$

Solving for the elevation angle between the ownship velocity vector and the z axis yields:

$$\text{Elo} = \arccos \frac{V_{zo}}{(V_{xo}^2 + V_{yo}^2 + V_{zo}^2)^{1/2}}$$

Using the same procedure above for the target, the elevation angle between the target and z axis is expressed as:

$$\text{Elt} = \arccos \frac{Z_t}{(X_t^2 + Y_t^2 + Z_t^2)^{1/2}}$$

The elevation angle is the difference of these two angles:

$$\text{El} = \text{Elo} - \text{Elt}$$

### Chapter Summary

These two coordinate systems fully describe the MTT environment. The interrelationship between the systems enables the targets to be located with respect to the ownship and also with respect to the earth. The ownship

gauges its altitude by measuring its height above the earth. As for the azimuth and elevation angles, these measurements are made with respect to the spherical system with the ownship as the origin in this system. As shown, both reference systems are indeed needed for this analysis.

### CHAPTER THREE - DEFINING THE TARGET

A target is a physical entity that a radar seeks to observe. The target exists independently of the radar, and is modeled by its own set of equations. In this chapter, the life of the target is addressed. It cannot be overemphasized that this is not a model of how the radar sees the target. It is a model of the target and it traces the target's existence through time.

#### Target Life Stages

A target goes through two phases in its life. It either exists and is termed confirmed, or it has ceased to exist and is termed deleted. Of course, the target can also be described in terms of its position and speed but these measurements are irrelevant to this chapter because position and speed figure more prominently into how the radar sees the target. Here, the concern centers around the target's existence which is a state of being that occurs whether or not the radar senses it.

#### Sequential Analysis for Target Confirmation and Deletion

Since it has been established that the life of a target is random in nature, it must therefore be modeled based on probabilistic theory. A method of sequential analysis is

applied to the problem. This is based on Blackman [2]. The sequential probability ratio test (SPRT) is used to choose between two hypotheses:

$H_0$  = no true targets are present so returns are from false alarms or clutter;

$H_1$  = A true target is present.

Every time data is received, one of three decisions must be made. One can accept  $H_0$ , accept  $H_1$  or wait until more data is available.

The probability that Hypothesis 1,  $H_1$ , exists is

$$P_{1k} = P_d^m (1 - P_d)^{(k-m)}$$

The probability that Hypothesis 0,  $H_0$ , exists is

$$P_{0k} = P_f^m (1 - P_f)^{(k-m)}$$

Where:

$P_f$  : Probability of detecting a false target

$P_d$  : Probability of detecting a true target

$m$  : number of detections

$k$  : scan time (sec)

#### Definition of the SPRT

The SPRT is defined as the ratio of  $P_{1k}$  to  $P_{2k}$ . It is written as:

$$U_k = P_{1k} / P_{2k}$$

This ratio is compared to two thresholds denoted as  $C_1$  and  $C_2$ . These thresholds are computed as follows:

$$C_1 = \frac{1 - \text{Beta}}{\alpha}$$

$$C_2 = \frac{\text{Beta}}{1 - \alpha}$$

$\alpha$  = probability of accepting  $H_1$  when  $H_0$  is true;

$\text{Beta}$  = probability of accepting  $H_0$  when  $H_1$  is true;

The decision logic is:

1. If  $U_k < C_1$ , accept  $H_0$
2. If  $U_k > C_2$ , accept  $H_1$
3. If  $C_1 < U_k < C_2$ , continue testing

With a little mathematical manipulation, a convenient set of expressions is derived as follows:

Take the natural logarithm of  $U_k$ :

$$\ln(U_k) = \ln(P_{1k}/P_{2k}) = (m) \ln \frac{P_d / (1 - P_d)}{P_f / (1 - P_f)} + (k) \ln \frac{(1 - P_d)}{(1 - P_f)}$$

$$\text{Let } a_1 = \ln \frac{P_d / (1 - P_d)}{P_f / (1 - P_f)}$$

$$\text{Let } a_2 = \ln \frac{(1 - P_f)}{(1 - P_d)}$$

$$\text{If } ST(k) = m(a_1)$$

$$\text{Then, } \ln(U_k) = ST(k) - k(a_2) .$$

The hypotheses can be redefined as follows:

$$\text{If } \ln(U_k) = ST(k) - k(a_2) < \ln(C_1), \text{ accept } H_0.$$

$$\text{If } \ln(U_k) = ST(k) - k(a_2) > \ln(C_2), \text{ accept } H_1.$$

Defining an upper and lower threshold:

$$\text{Let } Tu(k) = \ln(C_2) + k(a_2)$$

$$Tl(k) = \ln(C_1) + k(a_2)$$

Now, the decision logic can be defined in terms of these thresholds:

1. If  $ST(k) < Tl(k)$ , accept  $H_0$ .
2. If  $ST(k) > Tu(k)$ , accept  $H_1$ .
3. If  $Tl(k) < St(k) < Tu(k)$ , continue testing.

The upper and lower thresholds are modeled as two parallel lines with slopes equaling  $a_2$  and y-intercepts with values of  $\ln(C_2)$  or  $\ln(C_1)$  respectively. The test statistic,  $St(k)$ , increases by  $a_1$  each time a detection is made and remains unchanged if no detection is made. If the test statistic crosses the upper threshold, the target is confirmed. However, if it passes the lower threshold, it is deleted. This process is repeated for each target until a target is deleted. Figure 3.1 serves to illustrate this



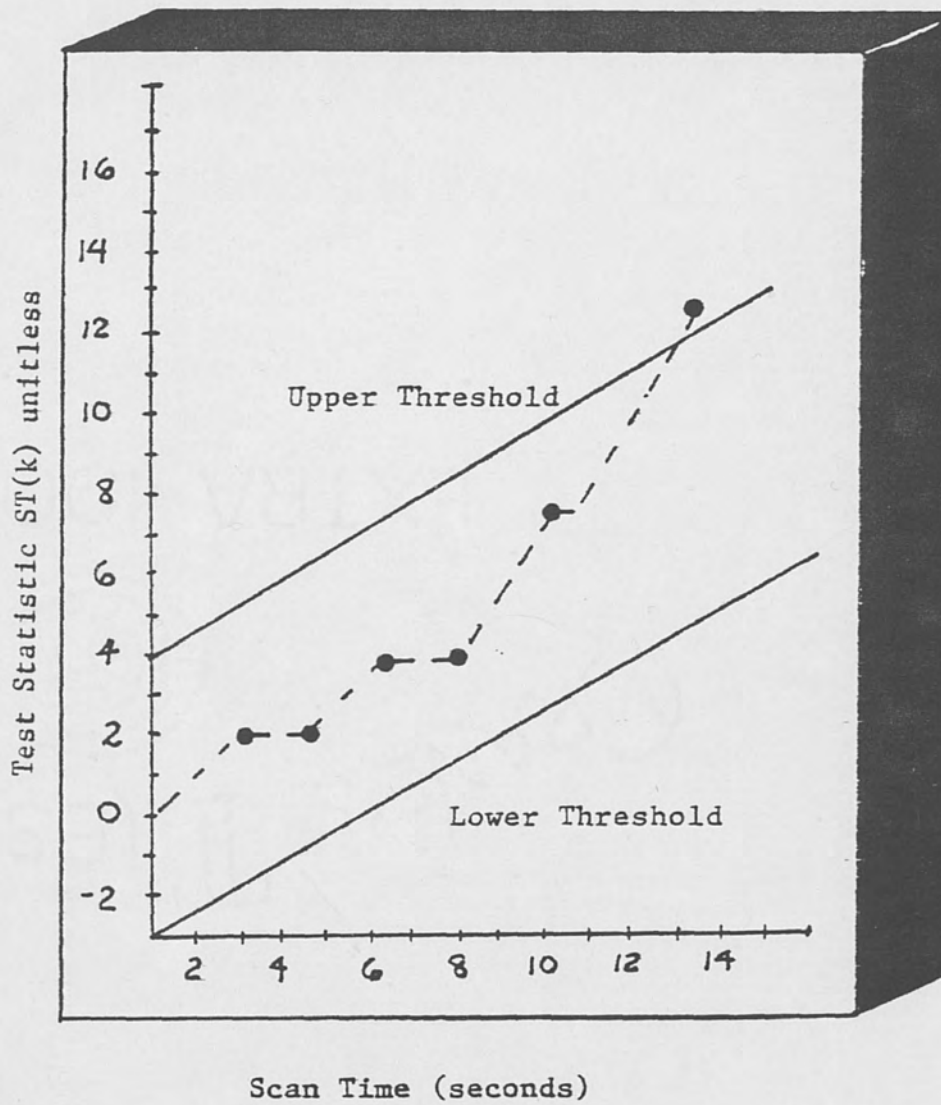


Figure 3.1 Threshold Detector Controlling the Targets' Lives.



principle. It is important to realize that this threshold detector bears no relation to the threshold detector explained in Chapter One. This chapter describes a statistical tool that monitors the life of the target.

#### Chapter Summary

To the radar, the target is a random phenomenon. When an observation is made, there is no guarantee that the target will remain on the radar's screen for any length of time. It can suddenly fall out of range, it can be shielded from view by mountains or buildings, it can be hidden by enemy countermeasures, or it may be perceived as being too small due to the squint angle on the antenna. At best the radar is satisfied with being able to monitor its behavior during its lifetime. This chapter provides a tool to effectively monitor each target's life.

## CHAPTER FOUR - DEFINING THE OBSERVATION

Any kind of data that the radar senses is referred to as an observation. An observation can be due to a single target, or to a cluster of targets, or it can originate from noise or clutter. Every observation is accompanied by a set of data corresponding to the object that was detected. This set of data consists of a range, range rate, azimuth angle and/or elevation angle measurements. These measurements provide clues as to the nature of the object that was detected on screen.

### Determining If an Observation Was Due to a Target

With each scan, the radar picks up a number of observations. How does it decide which ones are due to the target and which ones are due to noise alone? For each observation, the radar measures a certain range at which the target is located and a target cross section. These two factors are inserted into the radar range equation to obtain the signal-to-noise ratio. This ratio is then compared to the desired threshold level. If the obtained signal-to-noise ratio exceed the threshold level, the return is considered to be from a target. If it does not surpass the threshold level, the return is assumed to be from noise. This process was discussed in detail in Chapter One.

### Generating Target Positions From Observations

As stated above, the radar can take up to four kinds of measurements: range, range rate, azimuth angle and elevation angle. However, the radar knows that its receiver is haunted by noise and that the measured values are mere predictions of the actual position. Therefore, the radar processor incorporates the effects of noise into each of the four measurements to yield what is thought to be the true target position.

### Modeling the Life of an Observation

How long is the average life span of an observation and when and how does it cease to exist? The observation's life is modeled by the threshold detector described in Chapter One.

Assume an observation is correlated to a particular target, Target X. Also, assume it has been correlated to Target X for  $N$  consecutive scans. Suddenly, at the  $(N+1)$ th scan, the return due to Target X is not received. There are three major reasons this could have occurred. The first reason is that Target X has gone outside the detection range and can no longer be sensed. This corresponds to the situation in Chapter Three where a target falls below the lower threshold and is classified as being deleted. The second reason Target X is not detected is because although it is still within the antenna beam, it may be shielded by clutter or its cross section may appear too small due to the

squint angle. Thirdly, Target X may become so perfectly aligned with another target, Target Z, for example, that for a split second, these two targets appear as one, and one observation, either Observation X or Observation Z will not appear on screen for that scan.

Based on the above logic, it is not difficult to see why the corresponding observation is not immediately deleted. There is a good chance that Target X may reappear during the next scan. The system waits M scans (where M is usually two to three) before it deletes the observation.

#### Chapter Summary

It is obvious that it is impossible to cite an average life span of an observation. However, an observation has its own life span plus M more scans. This is to insure that the target being tracked was not just temporarily hidden from view.

## CHAPTER 5 - GATING

Gating is a technique whereby a boundary is formed around a predicted target position. The observation falling within that boundary or gate that is the closest to the target in question is correlated to that target. This is known as nearest neighbor correlation and is the first step in the association problem. Association refers to the process whereby the radar attempts to figure out with which target its returns should be paired. Gating eliminates unlikely associations by guaranteeing that observations falling outside the target's gate will not even be considered as a match because they are too far away. In situations where more than one observation lies within a particular gate, or where gates of closely spaced targets overlap, additional logic is utilized to solve the association problem. Figure 4.1 provides an illustrative explanation of this concept.

### Normalized Distance

In the gating procedure, a normalized distance is calculated based on the measured and predicted data. This



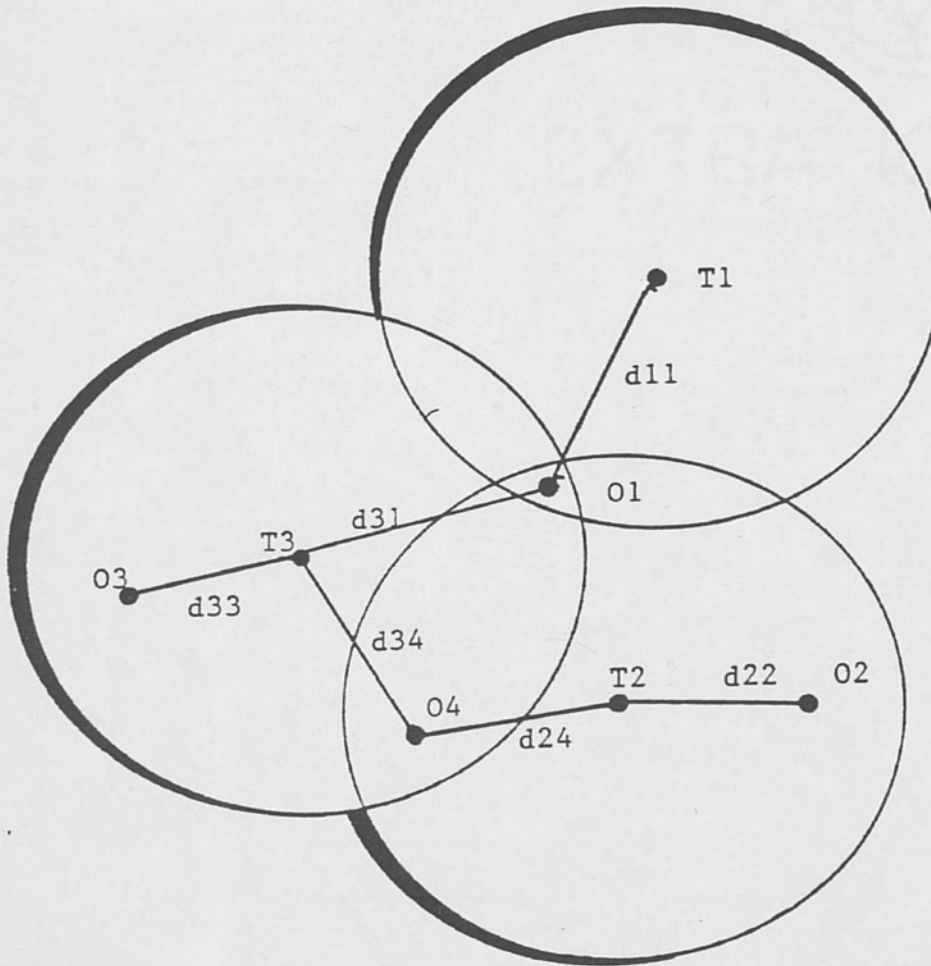


Figure 5.1 Example of Gating and Correlation.

distance is proportional to the physical distance from a target to each of the observations lying within that target's gate. It is computed as follows:

$$d^2 = \frac{(R_p - R_o)^2}{\text{Var}(R)} + \frac{(A_{zp} - A_{zo})^2}{\text{Var}(az)} + \frac{(E_{lp} - E_{lo})^2}{\text{Var}(El)} + \frac{(R_{dotp} - R_{doto})^2}{\text{Var}(Rdot)}$$

Where:

$R_p$  : Predicted target range (m)  
 $R_o$  : Observed target range (m)  
 $\text{Var}(R)$  : Variance of the difference,  $R_p - R_o$   
 $A_{zp}$  : Predicted target Azimuth angle (deg)  
 $A_{zo}$  : Observed target Azimuth angle (deg)  
 $\text{Var}(az)$  : Variance of the azimuth difference,  $A_{zp} - A_{zo}$   
 $E_{lp}$  : Predicted target elevation angle (deg)  
 $E_{lo}$  : Observed target elevation angle (deg)  
 $\text{Var}(el)$  : Variance of the elevation difference,  $E_{lp} - E_{lo}$   
 $R_{dotp}$  : Predicted target range rate  
 $R_{doto}$  : Observed target range rate  
 $\text{Var}(Rdot)$  : Variance of the range rate difference,  $R_{dotp} - R_{doto}$

If the normalized distance falls outside the specified boundary, the observation corresponding to that distance is immediately discarded as a possible match to the target in question.

### Modifications to the Normalized Distance Equation

The normalized distance equation presented above is presented for the most generalized case. Four sets of measurements can be used in computing the distance. Although this would tend to give the most accurate results, it is possible to obtain acceptable results by using fewer than four measurements. For example, one might design a system that only incorporates the range and the azimuth angle measurements in the association problem. Or, another system might depend entirely upon the range rate. It is important to know which measurements are crucial and which ones can be ignored. This is because the less measurements taken, the cheaper and less complicated the system will be.

## CHAPTER SIX - AN EXTENSION TO MUNKRE'S ALGORITHM FOR THE DYNAMIC CASE

Munkre's algorithm is a generic correlation technique. It is based on the principles of linear programming and operations research and attempts to find a "minimum cost" solution to a matrix problem. It was found that the radar correlation problem of associating targets to observations could be solved using Munkre's algorithm. Of course, a few modifications were required so that the algorithm could be applied to this radar system in particular. This chapter describes Munkre's algorithm in detail and then covers the modifications that were made to Munkre's algorithm.

### Matrix Format

The matrix formed for each iteration of Munkre's algorithm matrix is modeled in Figure 6.1. The targets denote the columns and the observations denote the rows. The elements of the matrix represent the normalized distances from target  $n$  to observation  $m$ . The object of the algorithm is to correlate the observations to the targets in such a way that the sum of the normalized distances is minimized. This is also referred to as nearest neighbor correlation. It implies that the target that is closest to an observation becomes correlated with it. Each target is correlated with only one observation. If there are more targets than observations,

	T1	T2	T3
O1	d11	d12	d13
O2	d21	d22	d23
O3	d31	d32	d33
O4	d41	d42	d43

Figure 6.1 Sample Matrix for Munkre's Algorithm.



the extra targets will not be correlated. Also, if the uncorrelated observations fall outside the gate around the target in question, they are not correlated with that particular target. Munkre's algorithm works best in a sparsely populated space. A few modifications were made to the algorithm to account for overlapping targets that clustered together and appeared as one large target. These changes are described in detail in this chapter.

### Munkre's Algorithm

Below is a word-for-word replica of Munkre's algorithm as presented by Bourgeois and Lassalle [3]. These steps refer to the matrix shown in Figure 6.1.

Preliminaries.

- (a) No lines are covered; no zeroes are starred or primed.
- (b) Let  $k = \min (\text{number of rows}, \text{number of columns})$ .
- (c) If the number of rows is greater than the number of columns, go to the last step (f) of the preliminaries.
- (d) For each row of the matrix ( $a_{ij}$ ) subtract the value of the smallest element from each element in the row.
- (e) If the number of columns is greater than the number of rows, go to Step 1.
- (f) For each column in the matrix, subtract the smallest element of the column from each entry in the column.

Step 1

- a. Find a zero,  $Z$ , of the matrix.

- b. If there is no starred zero in its row or its column, star the zero ( $Z^*$ ).
- c. Repeat Step 1(b) for all zeroes of the matrix.
- d. Go to Step 2.

#### Step 2

- a. Cover every column containing a starred zero ( $Z^*$ ).
- b. If  $k$  columns are covered, the locations of the  $Z^*$  form the row-column associations (observation-target pairs). The algorithm is now finished.
- c. Otherwise go to Step 3.

#### Step 3

- a. Choose an uncovered zero and prime it to ( $Z'$ ).
- b. If there is no starred zero in the row of  $Z'$ , go to Step 4.
- c. If there is a starred zero ( $Z^*$ ) in the row of  $Z'$ , cover this row and uncover the column of  $Z^*$ .
- d. Repeat Step 3 until zeroes are covered.
- e. Go to Step 5.

#### Step 4

- a. The sequence of alternating starred and primed zeroes is as follows:
  1. Let  $Z_0$  denote the uncovered  $Z'$ . If there is no  $Z^*$  in the column of  $Z_0$ , go to Step 4(a-6).
  2. Let  $Z_1$  denote the  $Z^*$  in the column of  $Z_0$ .
  3. Let  $Z_2$  denote the  $Z'$  in the row of  $Z_1$ .

4. Continue performing steps 4 (a-2) and 4 (a-3) (where in Step 4 (a-2) look in the column of  $Z_2$  instead of  $Z_0$ ) until a  $Z_2$  which has no  $Z^*$  in its column is found.
5. Unstar each starred zero of the sequence.
6. a. Star each primed zero of the sequence.
  - b. Erase all primes from primed zeros and uncover every line.
  - c. Go to Step 2.

#### Step 5

- a. Find the smallest uncovered element in the matrix and call it  $h$ ;  $h$  will be positive.
- b. Add  $h$  to each covered row.
- c. Subtract  $h$  from each uncovered column.
- d. Return to Step 3 without altering stars, primes or covered lines.

#### Derivation of Munkre's Algorithm

Munkre's algorithm is based on the following two Theorems summarized by [3]:

Theorem 1: Given a column vector  $(c_i)$  and a row vector  $(r_j)$ , the square matrix  $(b_{ij})$  with the elements:

$$b_{ij} = a_{ij} - c_i - r_j$$

has the same optimal assignment solution as the matrix  $(a_{ij})$ . Such  $(a_{ij})$  and  $(b_{ij})$  are said to be equivalent.

Theorem 2: Given a matrix,  $(a_{ij})$ , if  $m$  is the maximum number of independent zero elements in this matrix, then there are  $m$  lines (row or columns or both) which contain all the zero elements of  $(a_{ij})$ .

A set of elements of a matrix are independent if none of them occupies the same row or column. So,  $a_{12}$  and  $a_{21}$  are independent while  $a_{13}$  and  $a_{23}$  are not.

The preliminary step as well as Step 5 of Munkre's algorithm apply the principles of Theorem 1. For the preliminary step, the column vector is created by letting  $c_i$  = smallest element in the  $i$ th row of  $(a_{ij})$ . The row vector,  $r_j$  = smallest element in column  $j$  of the new matrix  $(a_{ij} = a_{ij} - c_i)$ . In Step 5,  $c_i = h$  or  $0$  depending on whether the  $i$ th row is covered or not. Also  $r_j = 0$  or  $-h$  depending on whether column  $j$  is covered or not.

Theorem 2 is applied in Steps 2 and 4. Here, a maximum set of independent zeroes and a minimum set of lines containing all zeros are found.

#### Adjusting Munkre's Algorithm to Support a Dynamic Environment

Munkre's algorithm, as presented in the literature, had to be modified to handle a multitarget dynamic environment. This refers to an environment where targets are in constant motion from scan to scan and where two or more targets can

cross and appear to the radar as a single target.

### Results of the Simulation

It was found that when targets remained separated in space, Munkre's algorithm repeatedly associated each observation with the intended target. However, the algorithm was not equipped to deal with a case where one target would overlap with another to form a single cluster. The algorithm interprets the environment through the normalized distance equation. If range is the only measurement available to the system for example, then two targets that are separated by the same distance in range will be considered a single target. This principle is based upon the fact that no two objects can simultaneously occupy the same block of space at exactly the same time. Although this principle is logical in every way, it can work both for and against the system. If one target generated two observations, this algorithm would realize that since only one target can occupy any particular space at one time, both observations were due to the same target rather than being due to two separate targets.

However, this logic can work against the system. The radar only processes the information available. For example, if the radar is only provided with range information, any two or more targets that are the same distance (or very close as in about five to ten meters ) away in range will be classified as overlapping and will be



seen as one target. In reality, the targets may be the same distance away in range but may be separated in angle or in range rate. Without additional information though, the radar concludes that one and not two targets exist. However, if two measurements are available, such as range and azimuth angle, then two targets will appear to overlap if they are not only the same distance away but form the same angle in the x-y plane. If three measurements are used, there leaves even less room for miscorrelation to occur. The number of measurements needed and the kinds of measurements necessary for acceptable results is discussed in the next chapter.

When two returns with equal values for the normalized distance were detected, the signal processor was incapable of solving the resulting matrix. This is because such a situation was not provided for in the algorithm. The situation described above was incorporated into the algorithm as follows: Step 3a of the steps in Munkre's algorithm reads:

Choose an uncovered zero and prime it to ( $Z'$ ).

This step assumes that an uncovered zero exists. No provisions were made for the case where all zeroes were covered but  $k = \min(\text{number of rows, number of columns})$  columns were not yet covered. Therefore, the processor remained at Step 3a, and no solution was ever made. This step in Munkre's algorithm was rewritten as follows:

## Step 3

a. Choose an uncovered zero and prime it to Z'.

1. If no uncovered zero is found, the algorithm is FINISHED!

What real life situation does this modification correspond to? If two of the normalized distances in the same column of Figure 6.1 have the same value, the algorithm as presented in [3] would reach any solution at all. This situation occurs when a radar perceives the returns as emanating from a single target rather than from two targets. The remaining steps of the algorithm were not modified.

Several examples follow. They trace the target through three scans. During the first scan, one sees two targets approaching. During the second scan, the targets cross and appear as one. On the following scan, the targets separate and form two targets once again. The impact of the modification is clearly depicted in these examples. These examples also serve to show how Munkre's algorithm works. For these three examples, assume the only available measurement is the range. The equation for the normalized distance for this case is:

$$d = \frac{\text{abs} \left[ \frac{(R_t - R_o)^{1/2}}{2} \right]}{(SDr)}$$

TABLE 1

RANGE MEASUREMENTS FOR THREE SCAN PERIODS FOR FOUR TARGETS

TARGET NUMBER	SCAN NUMBER		
	1	2	3
1	922.82	932.2	942.13
2	724.98	730.27	737.83
3	764.26	737.83	714.49
4	439.09	446.9	455.19

\* Note. All measurements are in units of meters.

TABLE 2

RANGE MEASUREMENTS FOR THREE SCAN PERIODS FOR  
FOUR OBSERVATIONS

OBSERVATION NO.	SCAN NUMBER		
	1	2	3
1	915.09	913.07	957.39
2	738.01	741.09	730.25
3	755.76	741.55	718.17
4	452.18	454.04	465.18

\* Note. All measurements are made in units of meters.

Where:

d : Normalized distance from target to observation  
 Rt : Range from target, t, to the ownship (meters)  
 Ro : Range from observation, o, to the ownship (meters)  
 SDr: Standard deviation of the range\*

\* For the examples that follow, SDr = 20.

#### EXAMPLE 1:

Drawing from the range measurements in Tables 1 and 2 for scan time 1, the initial matrix is set up as shown. For illustrative purposes, d11 is derived as follows:

From Table 1, at scan time 1, the distance from Target 1 to the ownship is 922.82 meters. The radar measured the distance to be 915.09 meters. The normalized distance is:

$$d = \frac{\text{abs} ( 922.82 - 915.09 )^2}{(400)} = 0.15$$

The remaining normalized distances are computed in a similar fashion. The matrices that follow are a direct implementation of steps in Munkre's algorithm listed earlier in this chapter.



## Initial Matrix (Scan time 1)

	T1	T2	T3	T4
O1	0.15	90.35	56.87	566.45
O2	85.39	0.42	1.72	223.38
O3	69.78	2.37	0.18	250.69
O4	553.76	185.05	243.49	0.45

## Preliminaries (Scan time 1)

	T1	T2	T3	T4
O1	0.00	90.2	56.72	566.3
O2	84.96	0.00	1.30	222.96
O3	69.60	2.19	0.00	248.32
O4	553.31	185.6	243.04	0.00

## Step 1 (Scan time 1)

\* If the normalized distance exceeds 100, it is denoted as X.

	T1	T2	T3	T4
O1	0.00*	90.2	56.87	X
O2	84.96	0.00*	1.30	X
O3	69.60	2.19	0.00	X
O4	X	X	X	0.00

## Step 2 (Scan time 1)

	C	C	C	C
	T1	T2	T3	T4
O1	0.00*	90.2	56.87	X
O2	84.96	0.00*	1.30	X
O3	69.60	2.19	0.00*	X
O4	X	X	X	0.00*

The capital C's above the column denote the which columns are covered. Since all columns are covered, the algorithm is complete. The following associations are made:

Target 1 => Observation 1

Target 2 => Observation 2

Target 3 => Observation 3

Target 4 => Observation 4

Looking at the ranges in Tables 1 and 2, the above associations were predictable. The targets and observations with the most similar range measurements were correlated.

During the second scan time, a look at Table 2 indicates that observations 2 and 3 appear to overlap, ie, their ranges are practically indentical. The positions for targets 2 and 3 however are actually separated by seven meters. But the radar does not realize this (as reflected in the observation measurements). Below is an example of what happens when the radar sees two targets overlap and how the modification to Munkre's algorithm solves the problem.

#### EXAMPLE 2.

Initial Matrix (Scan time 2)

	T1	T2	T3	T4
O1	0.92	83.53	76.77	543.07
O2	91.31	0.29	0.03	216.24
O3	90.87	0.32	0.03	216.91
O4	571.2	190.3	201.4	0.12

Preliminaries, Step 1, Step 2, and Step 3a (Scan time 2)

	C		C	C
	T1	T2	T3	T4
O1	0.00*	82.62	75.85	X
O2	91.28	0.27	0.00*	X
O3	90.84	0.28	0.00	X
O4	X	X	X	0.00*

In the second example, the second matrix would appear unsolved because not all columns are covered. However, due to the modification to Step 3a in Munkre's algorithm, the processing is in fact finished.

Target 1 => Observation 1

Target 2 => uncorrelated

Target 3 => Observation 2

Target 4 => Observation 4

Observation 3 => uncorrelated

Because understanding Example 2 is crucial to grasping the implications of the modification to Munkre's algorithm, it will be explained in detail. In the initial matrix of example 2, one notices that the normalized distance from target 3 to observations 2 and 3 are

identical. Because of this, the return from observations 2 and 3 are assumed to emanate from the same target. Therefore, only one of the two observations is correlated to the target while the other observation remains in existence but is not correlated during that scan period. Because these targets are assumed to be in constant motion, the two targets in question will have undoubtedly separated by the third scan. Example 3 covers this case.

#### EXAMPLE 3.

This example is the shows the results of the third scan period.

Initial Matrix.

	T1	T2	T3	T4
O1	0.58	120.51	147.5	630.5
O2	112.23	0.143	.6209	189.15
O3	125.38	0.965	0.034	172.9
O4	568.7	185.86	155.4	0.25



Preliminary Step, Step 1, Step 2 and Step 3 combined.

	C	C	C	C
	T1	T2	T3	T4
O1	0.00*	X	X	X
O2	X	0.00*	0.478	X
O3	X	0.931	0.00*	X
O4	X	X	X	0.00*

In this example, the following associations were made:

Target 1 => Observation 1

Target 2 => Observation 2

Target 3 => Observation 3

Target 4 => Observation 4

In example 3, the targets separated and the radar was once again able to discern two targets instead of one.

The above example only incorporated range into the normalized distance equation. In the second example, where the two targets appeared to overlap, an additional measurement such as the azimuth angle might have shown that although these targets are equally distant from the radar in range, they are separated in angle. The more measurements one incorporates into the distance equation, the more accurate the radar will be in associating targets with

observations. Example four incorporates the azimuth angle into the normalized distance equation giving the radar processor additional information to help it out in the association scheme. Example 4 uses the same range flight data as Example 2. Table 3 contains the measured data needed for this example. Note that the ranges are identical to those from scan 2 in Tables 1 and 2. They were included in Table 3 for the sake of completeness.

The normalized distance equation used in this example is the following:

#### EXAMPLE 4.

#### Initial Matrix

	T1	T2	T3	T4
O1	1.43	181.	648.	885.
O2	225.	1.25	169	274.
O3	691.	193.	0.05	244
O4	930.	260.	233.	0.21

TABLE 3  
RANGE AND AZIMUTH ANGLE MEASUREMENTS FOR ONE SCAN

N	TARGET		OBSERVATION	
	RANGE	AZIMUTH ANGLE	RANGE	AZIMUTH ANGLE
1	932.2	62.22	913.07	60.78
2	730.27	40.97	741.09	39.01
3	737.83	12.96	741.55	13.19
4	446.9	23.75	454.04	24.33

Note: The range is measured in meters.

The azimuth angle is measured in degrees.

N denotes the target and observation number.  
Preliminary Step, Step 1, Step 2, Step 3

	C	C	C	C
	T1	T2	T3	T4
O1	0.00*	X	X	X
O2	X	0.00*	X	X
O3	X	X	0.00*	X
O4	X	X	X	0.00*

The following associations are made:

Target 1 => Observation 1

Target 2 => Observation 2

Target 3 => Observation 3

Target 4 => Observation 4

Example 4 shows the benefits of including more than one measurement in the normalized distance equation. With the addition, of azimuth angle, the correct association was made.

#### Chapter Summary

This chapter served not only to define Munkre's algorithm, it also presented the modifications that were necessary to adapt this algorithm to a multiple target environment. Significant examples followed to illustrate this impact of this change. The following chapter uses this

chapter as a stepping stone to pinpoint which measurements are most important to this system.



## CHAPTER SEVEN - SIGNIFICANT MEASUREMENTS OF AN MTT SYSTEM

Four types of measurements were taken to determine which were most important to the performance of the MTT system. The four measurements considered were the range, the range rate, the azimuth angle, and the elevation angle. These measurements were incorporated into the normalized distance equation which had a direct impact on the correlation performance. Presented below are the results of the simulation.

### Assumptions

Before presenting the results, the radar parameters used in the particular example as well as the type of environment are discussed. The environment consists of multiple targets appearing and disappearing within the radar beam at random times. Figure 7.1 illustrates this environment. One sees that for this particular data set, the number of targets increases with time and reaches a maximum of 29 targets after seventeen seconds. This maximum is a constraint necessitated by the limitations of the software. Looking at the observations, one sees that as the aircraft flies through space, it picks up an increasing number of targets. This simulates a space where numerous targets are closing in on the radar or vice versa. Because the number of targets do not diminish with time, it is

TABLE 4  
RADAR SYSTEM PARAMETERS USED IN THIS EXAMPLE

PARAMETER	VALUE	UNITS
Transmit Power	100	watts
Receiver Gain	30	dB
Transmitter Gain	30	dB
Operating Frequency	35	GHz
Noise Figure	8	dB
S/N Losses	3	dB
IF Bandwidth	50	MHz
Ambient Temperature	273	K
Atmospheric Attenuation	0.3	dB/km

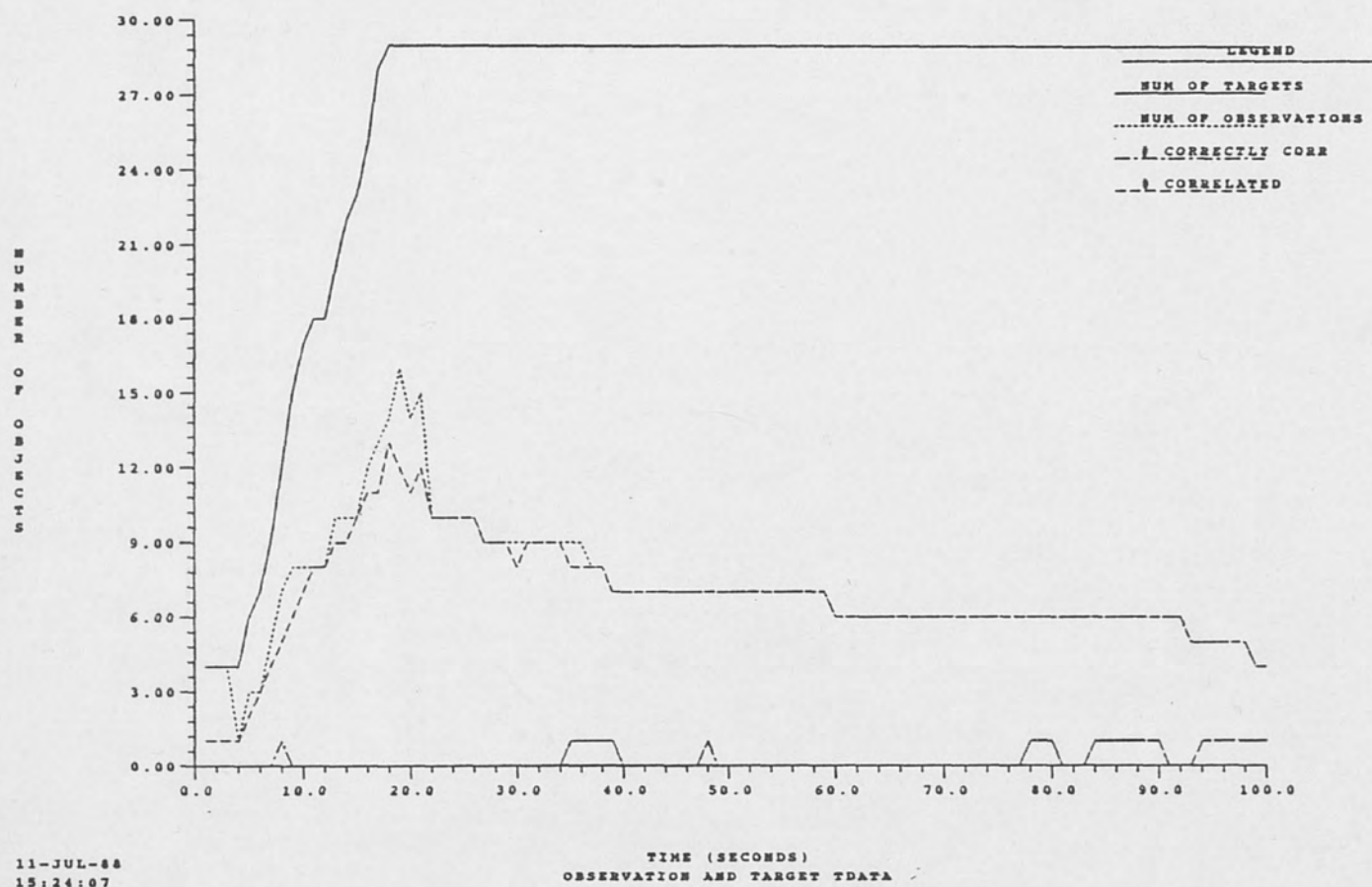


Figure 7.1 Tally of Number of Observations and Targets During 100 Seconds.

assumed that the aircraft did not destroy any targets. Instead, the targets' must have either flown out of range or have succeeded in shielding themselves. The system parameters characterizing the radar are listed in Table 4.

#### Normalized Correlation Ratio

Figures 7.2 through 7.8 illustrate the results of the simulation for one data set. These are plots of the normalized correlation ratio versus time. The normalized correlation ratio is the ratio of the number of correctly correlation observation to target pairs to the maximum number of correlations possible. If ten observations are made and thirty targets exist, the maximum number of correlations possible is ten. The other twenty targets were not seen by the radar and are therefore assumed not to exist. If there are thirty observations and only ten targets present, once again, the maximum number of correlations is still ten. The remaining twenty observations are assumed to be due to noise and or other sources. The closer the normalized correlation ratio comes to one, the better the correlation. Each figure is compared to the optimum case where four measurements are used.

#### Ranking the Measurements

Figures 7.2 to 7.8 are conveniently summarized in Table 5. This table ranks the measurement sets in order of importance. It can be seen from Figures 7.2 and 7.3 that

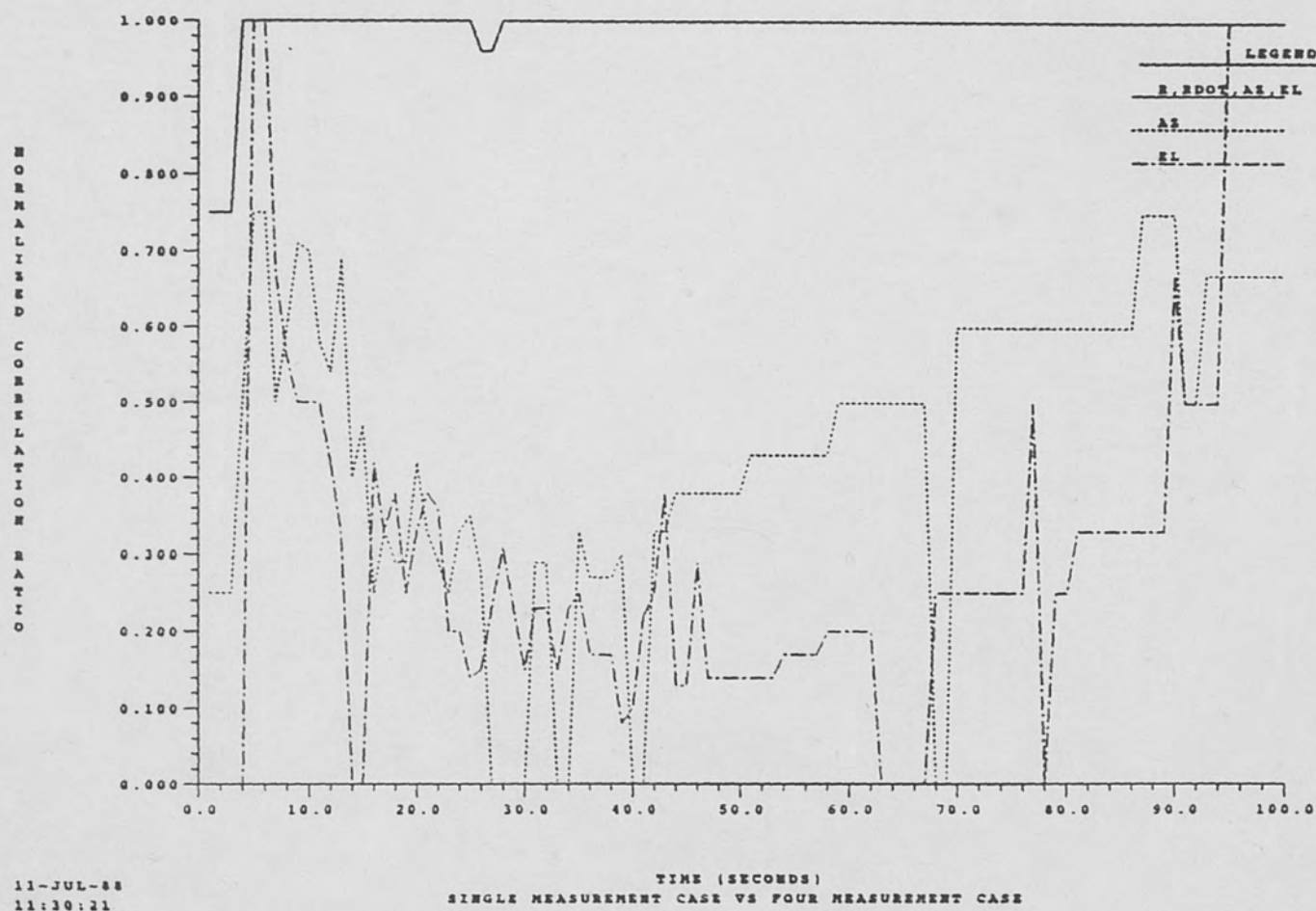


Figure 7.2 Correlation Plot 1.



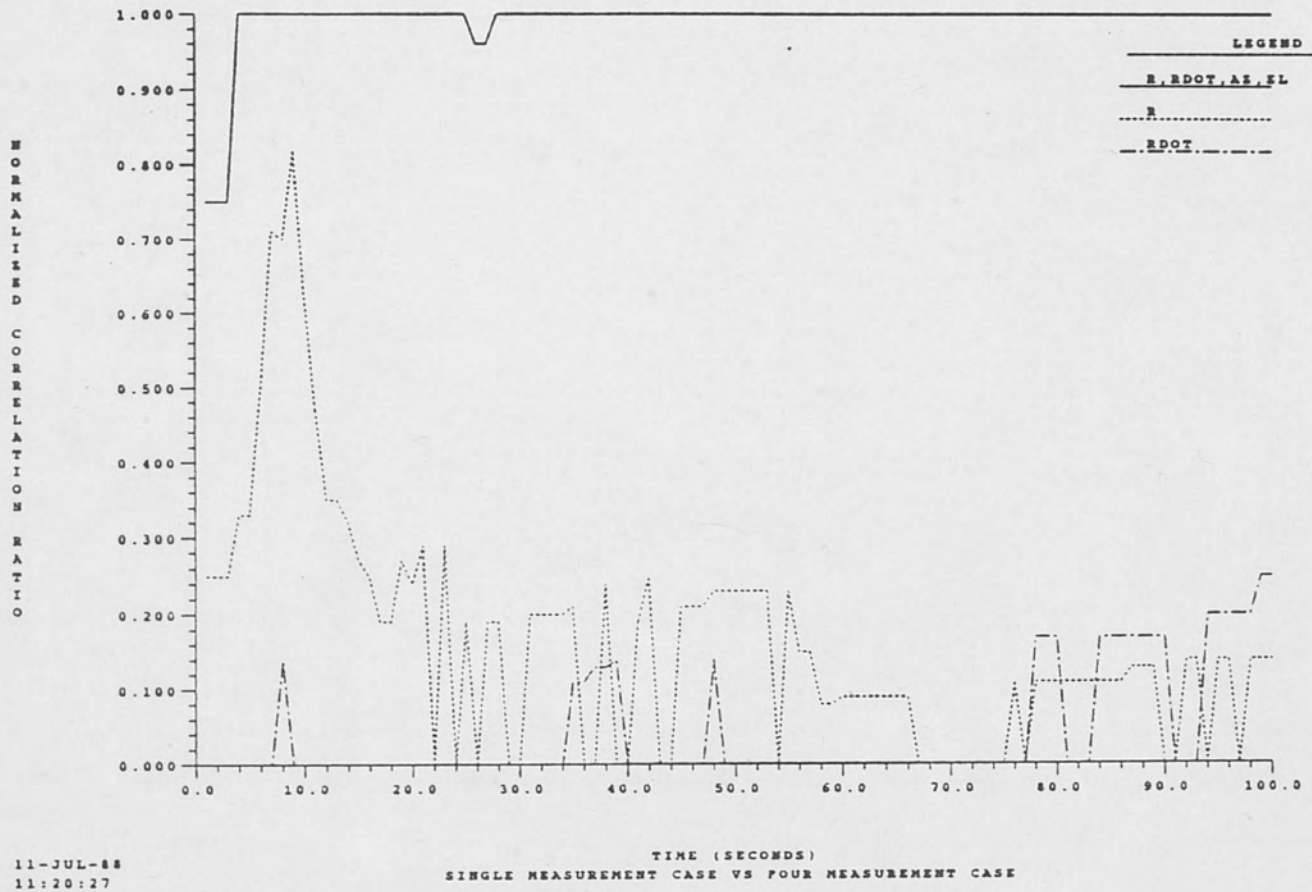


Figure 7.3 Correlation Plot 2.

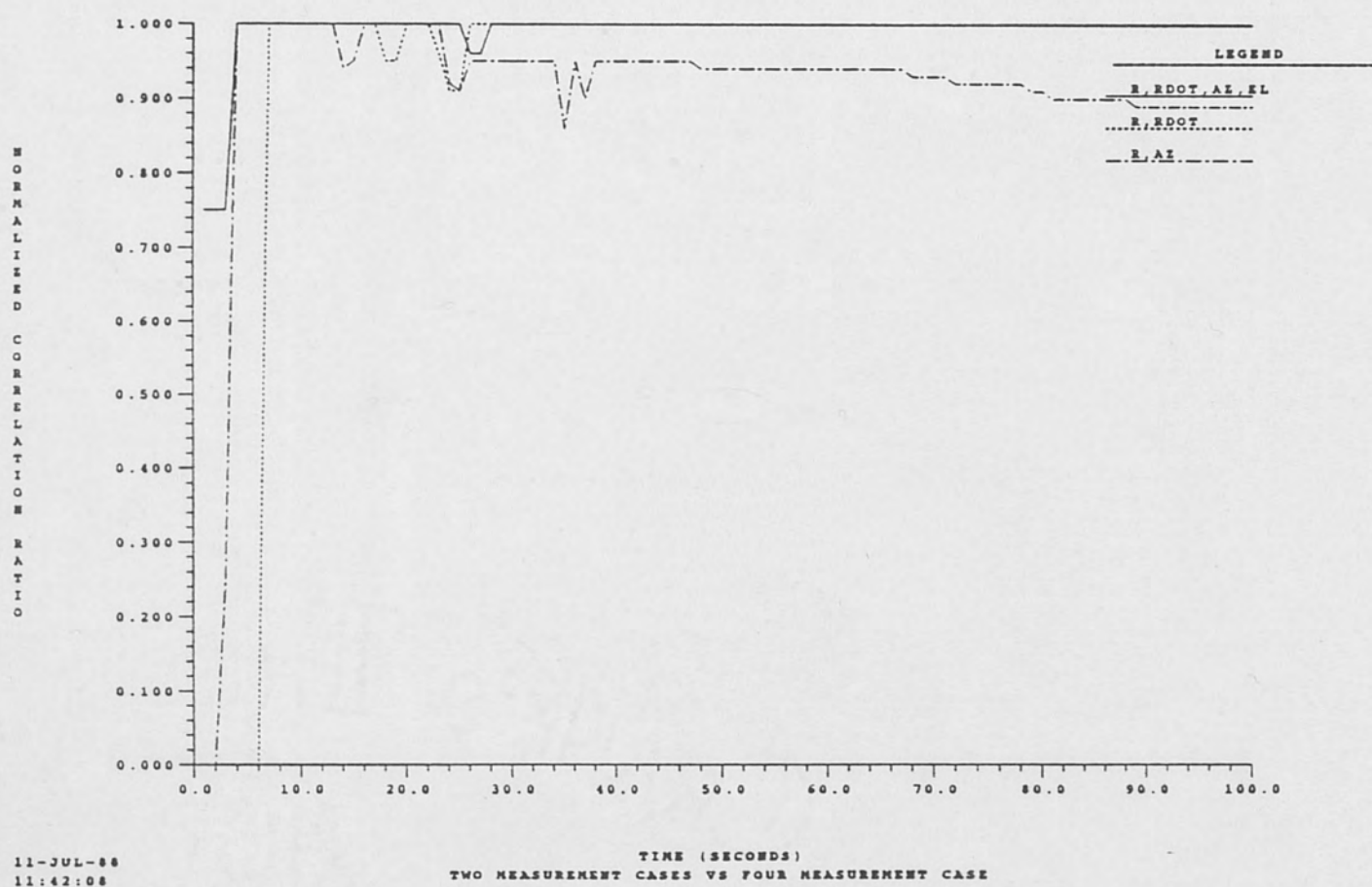


Figure 7.4 Correlation Plot 3.

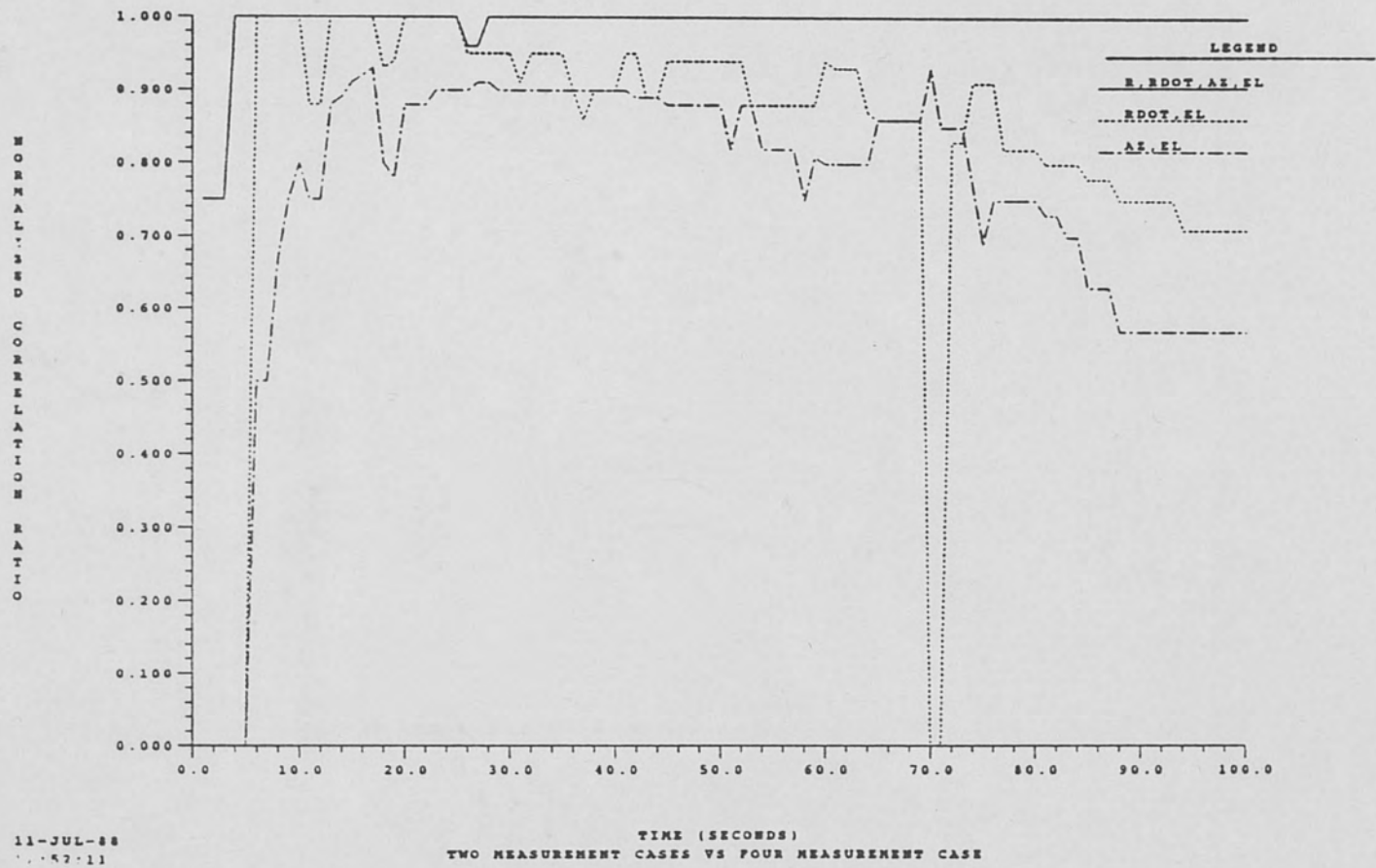


Figure 7.5 Correlation Plot 4.

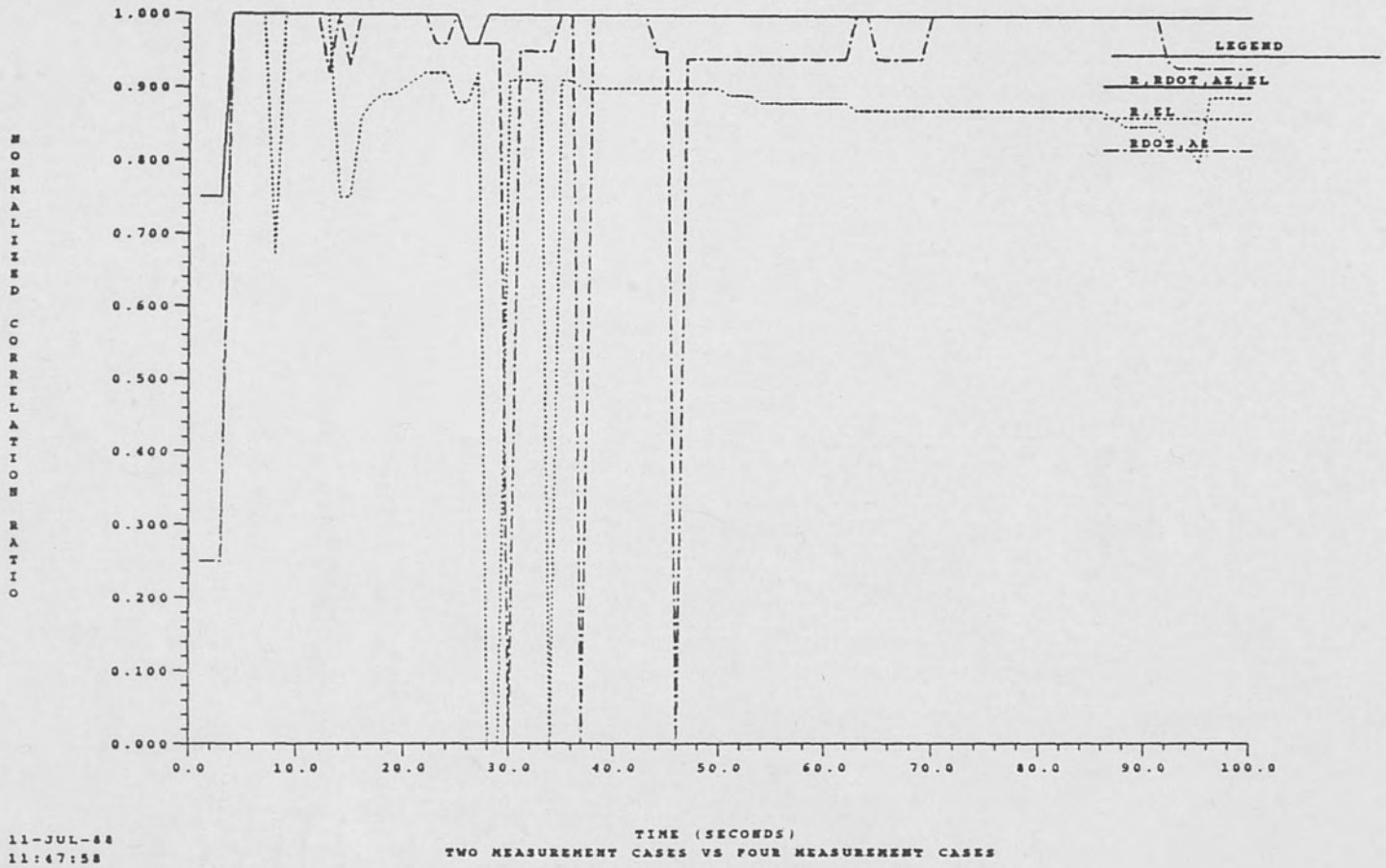


Figure 7.6 Correlation Plot 5.

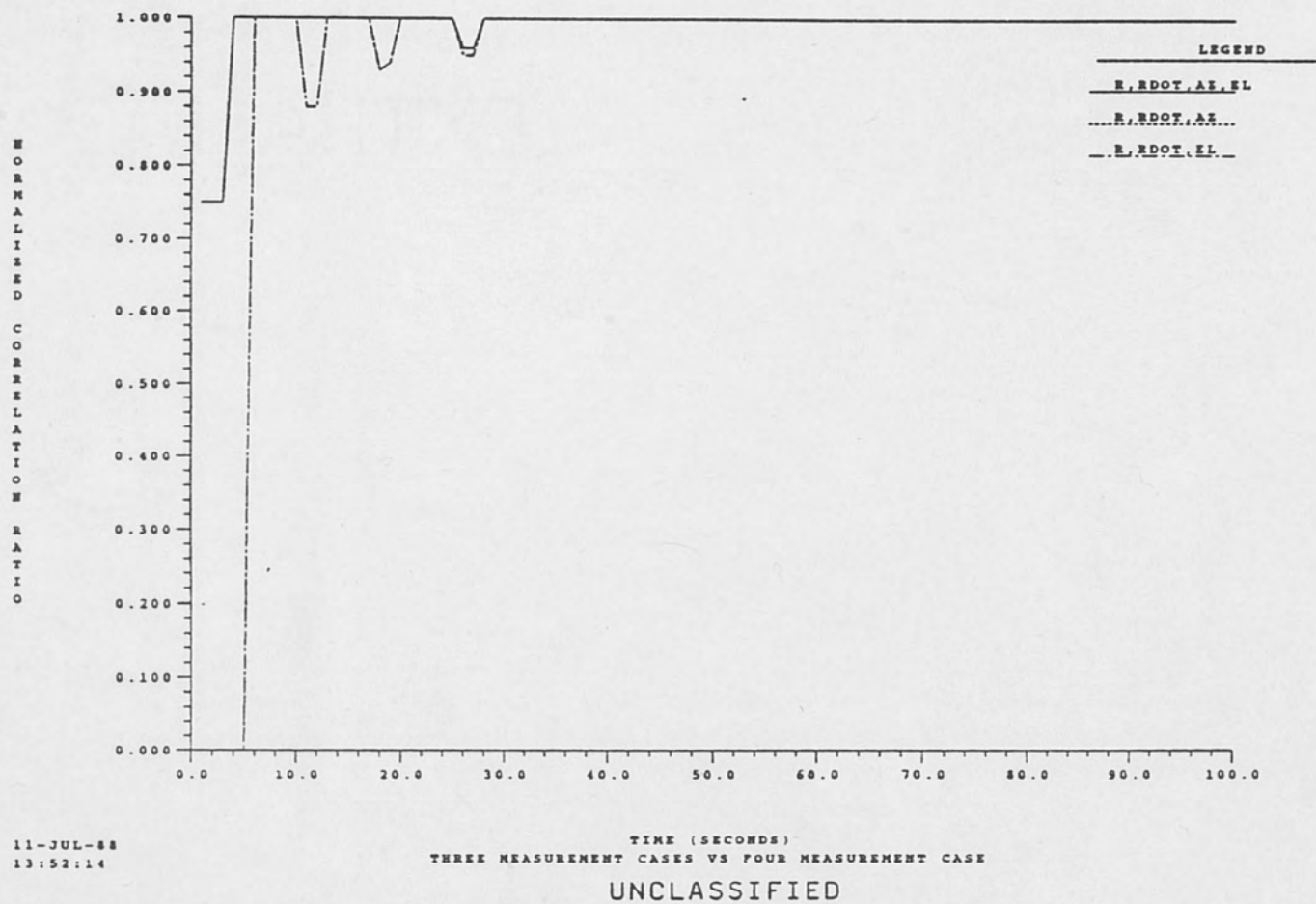


Figure 7.7 Correlation Plot 6.



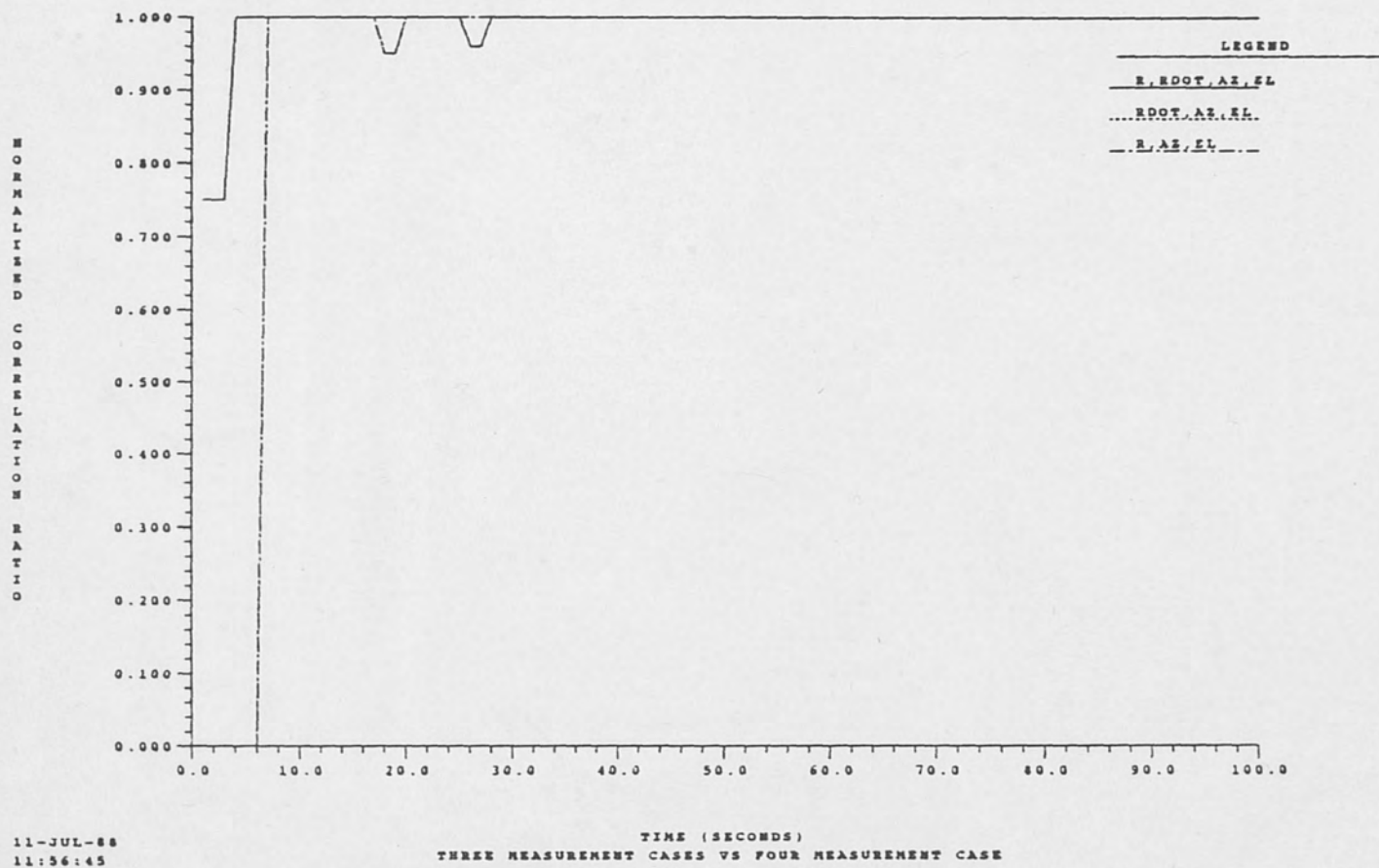


Figure 7.8 Correlation Plot 7.

TABLE 5  
SIGNIFICANT MEASUREMENTS OF AN MTT SYSTEM RANKED IN  
ORDER OF IMPORTANCE

RANKING	MEASUREMENTS
1	R, Rdot, Az, El
2	Rdot, Az, El R, Az, El R, Rdot, Az R, Rdot, El
3	R, Rdot
4	Rdot, El R, Az Az, El Rdot, Az R, El
5	Az El R Rdot

using two types of measurements drastically improved the system. As seen in Figures 7.4 through 7.6, most pairs resulted in an average correlation ratio bordering near 0.9. When three measurement types were used, the correlation ratio was close to 1.0 for all time.

### Results Discussed

As seen in the figures alluded to above, more than one measurement is necessary to obtain acceptable results. Using one measurement is out of the question for this system. The results obtained with two measurements are below the ideal case, although they yield an average correlation near 90 percent for all time. Depending upon the necessary accuracy, using only two measurements would be acceptable. For this case, it would appear that if one relied only upon the range and the range rate, the correlation would be close to 1.0 for nearly two thirds of the time. Three measurements give extremely close results to those obtained by four measurements. The needs and accuracy of a system being designed will dictate how many measurements will be used.

## CHAPTER EIGHT - THE DESIGN OF AN MTT SIMULATION

A simulation of a multiple target tracking system that utilizes the concepts and techniques of the first seven chapters has been developed. This simulation is written in Turbo Pascal, a Borland Product, and is designed to run on the IBM PC's and their compatible clones. The software is best characterized by its modularity. Not only is it broken down into over twenty procedures, it is further broken down into units. A procedure is a small segment of code with one primary task. A function is a segment of code usually written to define mathematical quantities not inherent to the system. An example would be the arctangent operation. It is not defined in Turbo Pascal. However, the arccos operation is defined. A function is written to express the arctangent in terms of the arccos. A unit consists of groups of procedures and functions all performing a similar task. Each procedure can be compared to a single brick who can stand alone if necessary but whose function is to work in conjunction with other bricks for a single cause. This analogy is represented in Figure 8.1. It is important to notice the order of the blocks. The initial procedure forms the ground floor because it provides direct information ( or support) to the brick above it and indirectly to every brick

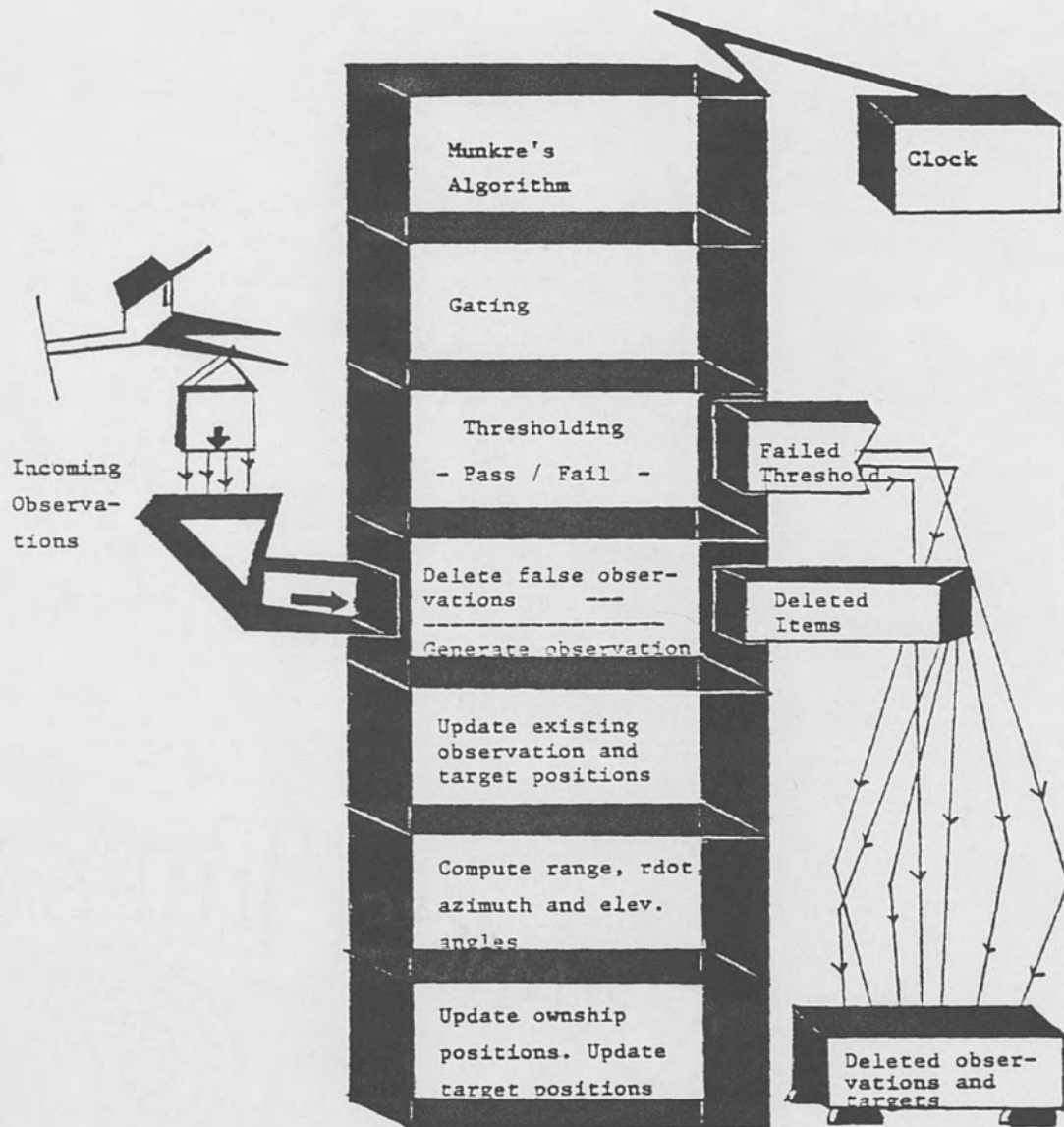


Figure 8.1 Block Diagram of the Simulation



above that one. The top brick, which represents Munkre's algorithm, depends upon the steps, or bricks, below for all its inputs. Following is a description of the simulation.

### Description of the Simulation

#### The Environment

This program simulates an environment as seen by an MTT TWS radar mounted on an aircraft, previously referred to as the ownship. All measurements are made with respect to the aircraft. The atmosphere is charged with targets, noise and clutter. Clutter is defined as an unwanted return disguising itself as a target. The aircraft is moving in an xyz space but receives data in a spherical space.

#### The Clock

The simulation is controlled by a real time clock. The sampling rate, a variable known as, TIME, is defined as the time it takes for the simulation to complete one full cycle. One full cycle consists of going through the block diagram in Figure 8.1 one time.

#### Updating Positions

The equations of motion are used to update the position of the ownship in the cartesian coordinate space. Subsequently, the positions of each of the existing targets is updated in the cartesian space as well. These equations are:

$$X_{\text{new}} = X_{\text{old}} + (V_x)(\text{TIME})$$

$$Y_{\text{new}} = Y_{\text{old}} + (V_y)(\text{TIME})$$

$$Z_{\text{new}} = Z_{\text{old}} + (V_z)(\text{TIME})$$

Where:

Xnew : The new x position (m)

X old : The old x position (m)

Vx : Velocity in the X direction (m/s)

TIME : Time increment (sec)

\* The same definitions hold for the y and z directions.

#### Conversion to the Spherical Coordinate System

Having updated the ownship position as well as the positions of the existing targets, range, range rate, azimuth angle, and elevation angle are computed from the updated cartesian coordinates. This conversion is explicitly described in Chapter Two.

#### Target Life

In any environment, the behavior of the target is random. It can dodge in and out of the antenna's beamwidth, or it can be destroyed by its enemy and in essence disappear into thin air. Thresholding, as described in Chapter Three, monitors the life of each target. In the software, the targets' lives are tracked via a status flag. If a target's existence is being checked, the status flag is set to zero. Once the target's existence has been confirmed, the flag is set to one, and the clock is reset for that target, at which

point the status flag is reset to zero. If a target is deleted, the status flag is set to 2.

#### Generate Observations

Given a target exists and given it has been correlated to an observation, that observation's position must be updated as well. Since an observation is defined as what the radar sees, range, range rate, azimuth and elevation angle are the parameters that are updated. Because an observation can be regarded as an approximation of the true target position, it is best approximated by adding random noise to the already updated target positions.

#### Dealing with Extra Observations

As the aircraft flies through the air, its radar scope is continuously receiving signals. Some are due to targets while others are due to clutter or noise. In addition to the existing observations which are paired to a target, and to the existing observations that are not yet correlated, a random number of observations are created during each clock cycle. This number can vary from zero to four observations. Because these are not due to targets known to exist at this point, measurements for the range, range rate, azimuth and elevation angle are generated randomly. The minimum allowable range is one hundred meters while the minimum allowable target cross section is 0.1 square meters. (These distances and number of random observations are defined for

this system and may not apply to other systems.)

### The Life of an Observation

An observation's life is just as ephemeral as the target's. It can appear on screen one second, and then disappear the next. This behavior must be simulated. Although this subject is explained in detail in the chapter on detection and in the chapter on observations, it will be explained here from a software point of view. There are two parameters in the radar range equation defined in Chapter One that are probabilistic in nature. These are the range and the target cross section. They are probabilistic in nature because they are due to the target and are not controlled by the radar. During each clock cycle, a range and target cross section is generated for those observations not associated with a target. A signal-to-noise ratio is computed and compared to a threshold. If the ratio exceeds the threshold, and the observation is not correlated to an existing target, then the return is attributed to a target and target coordinates are generated. This is done by adding random noise to the observed measurements. In order to update the target position as a function of time, it is necessary to determine the corresponding  $x, y, z, v_x, v_y,$  and  $v_z$  parameters in the Cartesian system from those in the spherical system. The velocity cannot be determined after one scan because velocity is defined as the change in position with time. At one instant of time, there is no

movement. Therefore, two clock cycles are needed to determine this. The measurements that the radar sees, ie range, range rate, azimuth and elevation angle are with respect to the ownship. In converting to the cartesian system, the x, y, and z parameters will be the distances from the target to the ownship. The equations of motion, shown above, are defined with respect to the xyz coordinate system centered at (0,0,0). Provisions are made in the software to account for this. If an existing observation is not correlated to a target after N number of scans, where N typically ranges from two to four, then that observation is deleted.

#### Gating

The normalized distance for each of the targets and observations is computed. The normalized distance may or may not include all four measurements taken by the radar. If a target or observation is deleted, the normalized distance is defined as -1. If the normalized distance exceeds a gate limit, normally set to 60, the distance is increased to 9001 such that it will not figure into the calculations.

#### Munkre's Algorithm

The previous steps have served as events forshadowing the climax: correlation of targets to observations via Munkre's algorithm. As discussed, certain steps were



modified in order to accomodate for a dynamic environment. The simulation is set up such that if desired, the matrix produced by Munkre's algorithm will echo to the CRT.

It must be stated that this software is the property of Martin Marietta and cannot be reproduced or modified without permission. However, it can be used by all interested parties.

## CHAPTER NINE - CONCLUSION

This thesis modified Munkre's algorithm and drew conclusions concerning the most significant measurements of a multiple target tracking system. It was found that it was impossible to isolate one measurement in particular and rank it as being most important. However, groups of measurements could be ranked in order of importance. Single measurements yielded very poor results. Pairs of measurements fared much better while groups of three types of measurements almost paralleled the optimum case of four measurements.

This thesis not only presents modifications to Munkre's algorithm, it has the effect of drawing the curious into a deep abyss taunting them to dive deeper into the subject matter by offering a wide opportunity of new and unanswered questions. For example, additional measurements could be included in the normalized distance equation. One such measurement is the friend or foe signal emitted by modern radar. When an aircraft spots an enemy, it sends out a signal and waits for a response that serves to identify the flying object under observation. No response usually results in the destruction of the target by the inquiring aircraft. Another area open for further study is the problem of crossing targets. This thesis relies on the nearest neighbor rule, and on the modification to Munkre's algorithm to handle the case of crossing targets. However,

once the targets have crossed, no attempt is made to predict the future position of the targets. This is in itself a thesis. Different types of gating equations can be used and a study can be done comparing the performance of several types of gating using this thesis' updated version of Munkre's algorithm. Dealing with clusters of targets is yet another facet of this problem that can be developed.

This thesis serves not only to present a modification to Munkre's algorithm for cases when targets cross, it also serves to whet a reader's appetite and to urge him or her to use this paper as a stepping stone in building the radar. An entire radar simulation takes years to build and over a dozen engineers to design. This thesis presents several building blocks of that design. However, given the significant modification to Munkre's algorithm, many new areas are now open to exploration.

## REFERENCES

1. Skolnik, Merrill. Introduction to Radar Systems. New York, New York: McGraw-Hill Book Compnay, 1980.
2. Blackman, Samuel. Multiple Target Tracking with Radar Applications. Norwood, Massachusetts: Artech House, 1986.
3. Bourgeois, F., and J.C. Lassalle. An Extension of the Munkres Algorithm for the Assignment Problem to Rectangular Matrices. Communications of the ACM, (Dec 1971), 802-804.
4. Bourgeois, F. and J.C. Lassallle. Algorithm for the Assignment Problem. Communications of the ACM, (Dec 1971), 805-806.
5. Bronson, Richard. Matrix Methods. Orlando, Florida: Academic Press, 1970.
6. Kaufmann, Arnold. Integer and Mixed Programming Theory and Applications. New York, New York: Academic Press, 1977.
7. Murty, Katta. Linear and Combinatorial Programming. New York, New York: John Wiley and Sons, 1976.
8. Silver, Roland. An Algorithm for the Assignment Problem. Communications of the ACM, (Nov 1960), 605-606.
9. Stimson, George. Introduction to Airborne Radar. El Segundo, California: Hughes Aircraft Companym 1983.
10. Stremler, Ferrel. Introduction to Communication Systems. Reading, Massachusetts: Addison-Wesley Publishing Company, 1982.